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Third Semester B.E. Degree Examination, Dec.2015/Jan.2016

Advanced Mathematics – II

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

- 1 a. Express the following in the form $a + ib$,
 $\frac{3}{1+i} - \frac{1}{2-i} + \frac{1}{1-i}$ and also find the conjugate. (06 Marks)
- b. Show that $(a + ib)^n + (a - ib)^n = 2(a^2 + b^2)^{n/2} \cos(n \tan^{-1}(b/a))$. (07 Marks)
- c. Find the fourth roots of $1 - i\sqrt{3}$ and represent them on an argand plane. (07 Marks)
- 2 a. Find the n^{th} derivative of $\cos 2x \cos 3x$. (06 Marks)
- b. If $y = e^{a \sin^{-1} x}$ then prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$. (07 Marks)
- c. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$. (07 Marks)
- 3 a. Find the angle between the radius vector and the tangent to the curve $r = a(1 - \cos \theta)$ at the point $\theta = \frac{\pi}{3}$. (06 Marks)
- b. Find the pedal equation to the curve $r = a(1 + \cos \theta)$. (07 Marks)
- c. Obtain the Maclaurin's series expansion of the function $e^x \sin x$. (07 Marks)
- 4 a. If $u = e^{x^2+y^2}$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \log u$. (06 Marks)
- b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$. (07 Marks)
- c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, find $J\left(\frac{u, v, w}{x, y, z}\right)$. (07 Marks)
- 5 a. Obtain the reduction formula for $I_n = \int_0^{\pi/2} \cos^n x dx$ where n is a positive integer. (06 Marks)
- b. Evaluate: $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} xy dy dx$. (07 Marks)
- c. Evaluate: $\int_0^1 \int_0^1 \int_0^1 (x + y + z) dx dy dz$. (07 Marks)

- 6 a. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (06 Marks)
- b. Evaluate: $\int_0^4 x^{3/2}(4-x)^{5/2} dx$. (07 Marks)
- c. Evaluate: $\int_0^{\infty} x^6 e^{-3x} dx$. (07 Marks)
- 7 a. Solve: $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$. (06 Marks)
- b. Solve: $(e^y + y \cos xy)dx + (xe^y + x \cos xy)dy = 0$. (07 Marks)
- c. Solve: $x^2 y dx - (x^3 + y^3)dy = 0$. (07 Marks)
- 8 a. Solve: $\frac{d^3 y}{dx^3} - 6 \frac{d^2 y}{dx^2} + 11 \frac{dy}{dx} - 6y = 0$. (06 Marks)
- b. Solve: $(D^2 - 4)y = e^x + \sin 2x$. (07 Marks)
- c. Solve: $(D^2 + D + 1)y = 1 + x + x^2$. (07 Marks)
